ABSTRACTS FOR RESEARCH TALKS AT THE 2024 JUNIOR WORKSHOP IN SEVERAL COMPLEX VARIABLES

Geometry of bounded domains in $\mathbb{C}^n$ via Statistical models

Gunhee Cho
University of California, Santa Barbara

The family of 1-dimensional normal distributions reveals a distinct differential geometric structure known as the Poincaré Upper Half Plane, arising from the Fisher-Information metric. The widely applied Cramér-Rao lower bound can be understood as a reflection of the Fisher-Information metric's decreasing nature. Furthermore, this metric's decreasing property resonates with the differential geometric interpretation of the Schwarz-Pick lemma within the Poincaré Disk of complex analysis. In this presentation, our goal is to extend the insights from the Poincaré Disk to encompass bounded domains in $\mathbb{C}^n$ as statistical models, leveraging observations from normal distributions and the Schwarz-Pick lemma. By treating bounded domains as statistical models, we establish the Bergman metric and the Fisher-Information metric are the same. Additionally, the establishment of a comparison between Bergman metric and another metric induced by Fisher Information metric corresponds to the Cramer-Rao inequality and identification of sufficient statistics. This recent work is a collaborative effort with J. Yum.

The local CR embedding problem

Sean Curry
Oklahoma State University

I'll talk about recent work with Peter Ebenfelt on the problem of realizing an abstract strongly pseudoconvex CR manifold of CR dimension at least 2 as a hypersurface in complex Euclidean space.

Rank Problems in Several Complex Variables

Dusty Grundmeier
Ohio State University

Let $r(z, \bar{z})$ be a real polynomial. The rank of r is given by the rank of the underlying matrix of coefficients. A natural problem is to study the rank of $r(z, \bar{z})\|z\|^2$. In this talk, we will discuss possible the ranks in several scenarios, including when $r(z, \bar{z})\|z\|^2 = \|h(z)\|^2$ for some holomorphic polynomial h. We will also describe an application to the degree estimates problem.

The Diederich–Fornæss index and the $\bar{\partial}$-Neumann problem

Bingyuan Liu
University of Texas Rio Grande Valley

A domain $\Omega \subset \mathbb{C}^n$ is said to be pseudoconvex if $-\log(-\delta(z))$ is plurisubharmonic in Ω, where δ is a signed distance function of Ω. The study of global regularity of $\bar{\partial}$-Neumann problem on bounded pseudoconvex domains is dated back to the 1960s. However, a complete understanding of the regularity is still absent. On the other hand, the Diederich–Fornæss index was introduced in 1977 originally for seeking bounded plurisubharmonic functions. Through decades, enormous evidence has indicated a relationship between global regularity of the $\bar{\partial}$-Neumann problem and the Diederich–Fornæss index. Indeed, it has been a long-lasting open question whether the trivial Diederich–Fornæss index implies global regularity. In this talk, we will introduce the backgrounds and motivations. The main theorem of the talk proved recently by Emil Straube and me answers this open question for $(0, n-1)$ forms.

A d-bar Carleman inequality and application to unique continuation

Ziming Shi
University of California, Irvine

We derive a Carleman type inequality of the d-bar operator, and as a consequence establish a unique continuation result for the differential inequality $|\overline{\partial}u| \leq V|u|$. Our method is based on a 1990 paper of T. Wolff. The proof can be divided into three parts: estimate of the Taylor series remainder of the fundamental solution, oscillatory integral on the sphere, and derivation of Carleman inequality using the osculation of the weight function.

Manifolds with Bergman metrics of constant holomorphic sectional curvature

John Treuer
University of California, San Diego

Xiaojun Huang and Song-Ying Li considered the problem of classifying complex manifolds whose Bergman metrics have constant holomorphic sectional curvature equal to c, where c can be either a negative constant, a positive constant or zero. In this talk, we build on their work and show a complex manifold with a Bergman space that is base-point free, separates directions and separates points cannot have identically zero holomorphic sectional curvature.

Curvature formulas associated to families of Hilbert spaces

Pranav Upadrashta
Stony Brook University

A result of Berndtsson states that the Chern connection associated to the metric on certain infinite rank vector bundles has positive curvature in the sense of Nakano. Following the ideas of Lempert and Szöke, together with Varolin we developed a framework to define the curvature of families of Hilbert spaces that might not fit together to form a holomorphic vector bundle. We will see this framework in action and obtain curvature formulas for metrics on families of Hilbert spaces associated to a general holomorphic submersion and a holomorphic hermitian vector bundle. In the first part we work with spaces of twisted harmonic forms associated to a proper submersion. In the second part, we work with Bergman spaces associated to a submersion whose fibers are domains. Additionally, we establish a lower bound on the curvature, from which we can recover Berndtsson's aforementioned result

The Cauchy-Riemann problem via extension operators

Liding Yao
Ohio State University

The Cauchy-Riemann problem, also known as the $\overline{\partial}$-problem, is a central problem in several complex variables. It concerns the regularity estimates to the equation $\overline{\partial}u = f$ on forms in a bounded domain $\Omega \subset \mathbb{C}^n$. I will talk about the background of the $\overline{\partial}$-theory and our recent works using new technique from extension operators. We use the so-called Rychkov's extension operator, which extends functions on a bounded Lipschitz domain and has boundedness on all Besov spaces and Triebel-Lizorkin spaces.

Optimal $\bar{\partial}$ regularity on product domains and its application to the Hartogs triangle

Yuan Zhang
Purdue University Fort Wayne

The $\bar{\partial}$ problem is to study the existence and regularity of the Cauchy-Riemann equation $\bar{\partial}u = f$ on pseudoconvex domains. Since Hörmander's fundamental L^2 theory, there has been substantial investigation for domains exhibiting favorable geometry and/or regularity. In this talk, we shall first focus on the $\bar{\partial}$ problem on a specific type of Lipschitz domains – product domains, and discuss recent advancements regarding its optimal Sobolev and Hölder regularity. Then we explore its application to the optimal Sobolev regularity on the Hartogs triangle. Part of the talk is based on joint works with Yifei Pan.

CR transversality of holomorphic maps between real hypersurfaces

Weixia Zhu
University of Vienna

Given two smooth generic submanifolds $M \subset \mathbb{C}^{n+d}$ and $M' \subset \mathbb{C}^{n'+d'}$ (with CR dimensions n and n', respectively), let H be a holomorphic map from an open neighborhood U of a given point $p \in M$ in $\mathbb{C}^{n+d}$ into $\mathbb{C}^{n'+d'}$ such that $H(M) \subset M'$. Letting $N = n + d$ and $N' = n' + d'$, H is said to be CR transversal to M' at p if

$$T^{1,0}_{H(p)}M' + dH(T^{1,0}_p\mathbb{C}^N) = T^{1,0}_{H(p)}\mathbb{C}^{N'}.$$

According to this definition, a CR map being CR transversal at $p \in M$ is equivalent to the derivative of its normal components at p along the normal direction being of full rank. When both the target and source manifolds are strictly pseudoconvex hypersurfaces, CR transversality always holds due to the classical Hopf lemma.

In this talk, I will discuss the history and established results concerning the CR transversality problem, and share our recent modest advancements on real hypersurfaces when the target manifold is a hyperquadric. Specifically, we consider holomorphic maps F from Levi non-degenerate real hypersurfaces $M_\ell \subset \mathbb{C}^n$ to a hyperquadric $\mathbb{H}^N_\ell$ with the same signature ℓ and $N - n < n - 1$. We show that F is either CR transversal to $\mathbb{H}^N_\ell$ or maps a neighborhood of M_ℓ in $\mathbb{C}^n$ into $\mathbb{H}^N_\ell$. Our proof is based on a refinement of the work by Huang and Zhang for the case when $N - n < (n - 1)/2$. This is a joint work with Xiaojun Huang.